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INSTANTANEOUS VELOCITY PROFILE IN A WAVY FLUID FILM

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Results of measuring the instantaneous velocity profiles in a laminar fluid film in the presence of stationary waves of different shape on its surface are presented.

It is known [1] that the velocity profile in the flow of a smooth laminar fluid film along a vertical wall is subject to a parabolic law

$$\frac{u}{U}=2\left(\frac{y}{h}-\frac{y^2}{2h^2}\right), \tag{1}$$

where the velocity on the surface of a film of thickness h is computed by means of the formula

$$U=\frac{gh^2}{2\nu}. \tag{2}$$

Formulas (1) and (2) are verified by experimental results [2-4] with a high degree of accuracy (to 1%) for a smooth laminar film.

A parabolic velocity profile in the form (1) is used in computing the wavy flow of a film in the majority of theoretical papers [5] using the method of integral relations, where h is the local film thickness and the velocity on the surface U is an unknown function of the time t and the longitudinal coordinate x. The case of a nonparabolic velocity profile is examined in [6, 7].

Data available in the literature [8-10] on the mean velocity profiles of a wavy film are distinguished by the significant spread, which does not permit making a deduction about the validity of (1) and (2) for a wavy film. Thus, it follows from the results of the most complete research [10] that the mean velocity profile of a laminar wavy fluid film is parabolic

$$u=Ay-By^2, \tag{3}$$

but the form of the coefficients A and B, in contrast to (1) and (2), depends on the number Re and the fluid viscosity.

An analogous situation is observed in the literature on the results of measuring the surface velocity of a fluid film. As follows from [11], the measured values of the dimensionless surface velocity U_0/u_0 for a laminar wavy film fluctuate between 1.15 and 2.2, where $u_0 = Q/h_0$ is the mean mass flow rate; i.e., the question whether the values of U are greater or less for wavy flow as compared to a smooth film for which $U/u_0 = 1.5$ is not even clarified.

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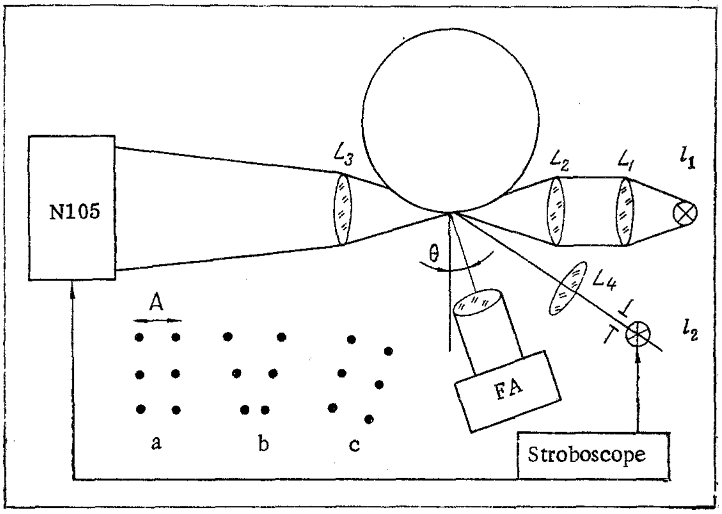

Fig. 1. Measurement scheme.

The main reason for the spread in the test data is the incorrect averaging of the instantaneous values of the measured velocities, since it is not known in what section of the wave the measurement is made. The fluid velocities in the film hence undergo noticeable changes along the length of the wave.

Experimental results on the instantaneous velocity profiles for a wavy fluid film are not found in the literature, which is apparently associated with the imperfection of the methods used.

We measured the instantaneous velocity profile in a fluid film in the presence of regular two-dimensional waves on its surface. The fluid film ran off along the outer surface of a vertical tube of 1 m length and 60.8 mm diameter. The tube was fabricated from stainless steel and polished to mirror brightness. The fluid in the working section was delivered through a 0.5-mm annular slot. The experiments were conducted with a water–glycerin solution of viscosity $\nu = 7.2 \cdot 10^{-6}$ m²/sec. The waves were excited by pulsations in the fluid discharge, just as in [12, 13]. The waves were strictly regular, stationary, and two-dimensional at a certain distance from the entrance. The velocity profile measurements were made in this domain.

In order to obtain an instantaneous velocity field in the wave, two methods were used and synchronized in this experiment: the shadowgraph method of determining the film thickness and the method of stroboscopic visualization of particles to measure the velocity. The shadowgraph method (Fig. 1) has been described earlier [13] and consists of recording the shadow cast by the fluid film on an N105 oscilloscope, under illumination by the lamp l_1 of the working section along the tangent to its surface. One of the authors developed the method of stroboscopic flow visualization and it was applied to fluid film flows in [8, 9]. The crux of the method is the following (Fig. 1). Production of circular aluminum particles several microns in size is initiated in the fluid. If the particles are photographed by the photo-apparatus (FA) under a side pulse of illumination by the lamp l_2, then an intermittent track of one particle is obtained on the photographic film, by means of which the particle velocity can be computed when the frequency and magnification factors are known. The frequency of the flashes by the lamp l_2 (IFK-120) is given by an audio frequency generator which triggers the stroboscope.

In the case of a mirror surface for the working section, the photographic apparatus will record not only the track of the real particle, but also the track of the imaginary image of the particle produced by the mirror when taking a photograph at the angle θ to the normal to the surface (Fig. 1), as is shown in Fig. 1a, b, and c. A formula to determine the distance of the particle from the wall

$$y = \frac{A}{2N \sin\theta} \sqrt{n^2 + \operatorname{tg}^2\theta\,(n^2 - 1)} \tag{4}$$

can be obtained from simple geometric constructions, where N is the magnification factor measured in the absence of fluid in the plane parallel to the film-frame plane, n is the fluid index of refraction, and A is the distance between the real and imaginary images of the particle on the photographic film.

The characteristic twin particle tracks for three flashes are shown in Fig. 1a, b, and c. The case a corresponds to a smooth film in the presence of only the longitudinal velocity component which is computed by means of the formula

$$u = \frac{x_{i+1} - x_i}{N} f, \tag{5}$$

where x_i is the longitudinal coordinate of the particle image on the photographic film for the i-th flash, and f is the frequency of the flashes. Case b corresponds to a smooth film, but there is still a transverse velocity component equal to

$$v = (y_{i+1} - y_i) f, \tag{6}$$

where y_i is the transverse coordinate of the particle, computed by means of (4). Case c corresponds to a wavy fluid film when the film surface is inclined at some angle to the longitudinal axis x.

An analysis of the profiles of all kinds of waves observed in the experiment showed that this angle reaches the maximum value of 23° only in the region of the leading wave front. Therefore, the simplified formulas (4)-(6) can be used, which results in an additional measurement error of not more than 6.5% for u and 3.5% for y in the region of the leading wave front. The main measurement error was 4% for u, 2% for the longitudinal coordinate x, and 7% for the transverse coordinate y, where the error in measuring x was referred to the wavelength λ.

The method described was checked on a freely running smooth fluid film with Re = 8-30. Measurements showed that the experimental velocity profiles are described with good accuracy by the Nusselt formula (1), (2).

Tests on a wavy film were conducted for Re = 5-14.5 and for three characteristic types of waves: cnoidal 1, intermediate 2, and rolling 3. The main characteristics of these waves are presented in Table 1 for Re = 12.4. Here c is the wave phase velocity; h^* and h_0 are, respectively, the maximum and mean film thickness; and U_0 is the velocity on the film surface, averaged with respect to the wavelength.

Certain results of measuring the instantaneous velocity profiles for waves of the types 1 and 3 (Table 1) are presented, respectively, in Fig. 2a and b. Here the numbers on the wave profiles denote the sections for each of which the velocity profile has been constructed.

Here 500-600 experimental points were obtained for each type of wave.

The domains where the transverse velocity component was determined in the experiment are denoted by the dashed curve near the wave; its maximum value can reach 0.5u. Curve I has been constructed by means of (1) for $U = u^*$ and $h = h^*$. For the rolling wave $u^* = c$.

As is seen from the graphs, the velocity profile changes slightly in the domain of maximum film thicknesses, but it undergoes abrupt changes in the domain of minimum values of h (sections 1, 9 in Fig. 2b and section 2 in Fig. 2a). The spread in the points reaches 100% here. The flow is purely laminar in the domain of the residual layer (sections 12 and 13 in Fig. 2a) and is described by the Nussel formula, curve II in Fig. 2a.

TABLE 1. Wave Characteristics of Two-Dimensional Waves for Re = 12.4

No.	Kind of wave	$\nu \cdot 10^6$, m^2/sec	h_0, mm	h^*, mm	λ, mm	c, mm/sec	$\frac{c}{u_0}$	$\frac{U_0}{u_0}$
1	Rolling	7,2	0,545	1,12	36	460	2,83	1,27
2	Intermediate	7,3	0,56	0,84	15	340	2,1	1,47
3	Cnoidal	7,1	0,56	0,73	11,8	310	1,93	1,48

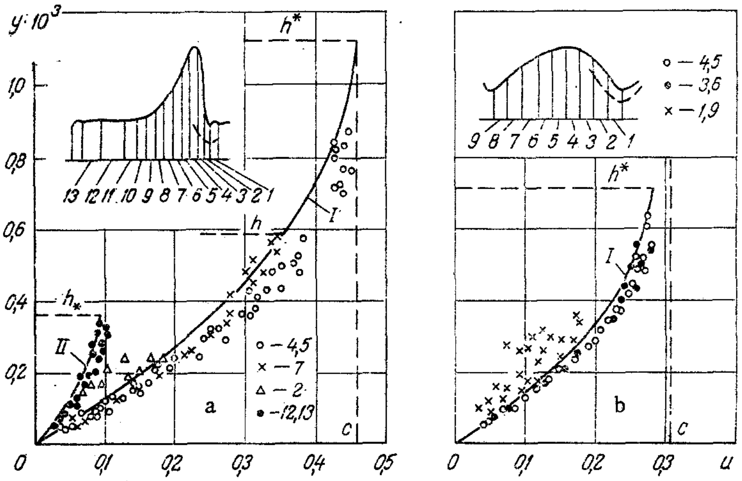

Fig. 2. Instantaneous velocity profiles in a wavy fluid film: I) formula (1) for $U = u^*$ and $h = h^*$; II) formula (1) for $U = gh^2/2\nu$ and $h = h_*$; $y \cdot 10^3$, m; u, m/sec.

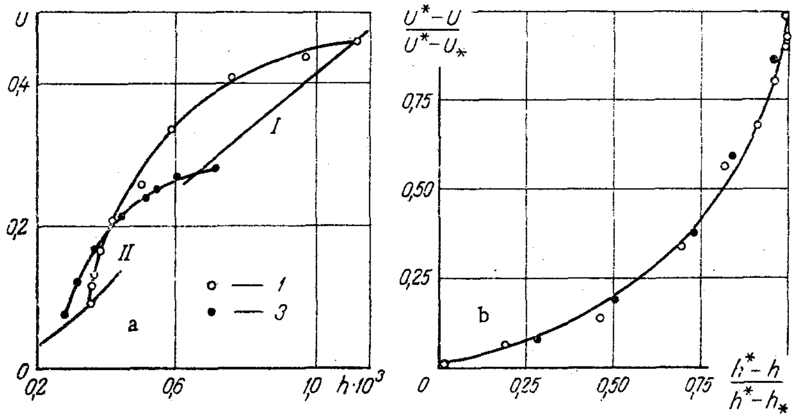

Fig. 3. Velocity on the wave surface: I) $U = c = kh^*$ [13]; II) $U=gh^2/2\nu$; see Table 1 for definition of 1 and 3; $h \cdot 10^3$, m.

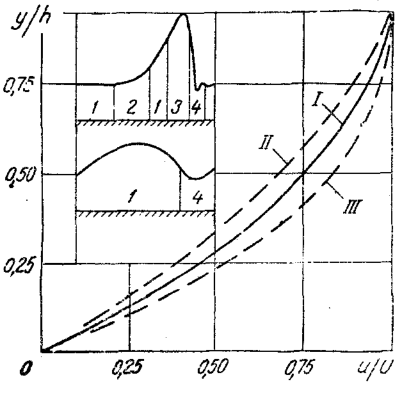

Fig. 4. Dimensionless velocity profile.

The dependence of the surface velocity U on the film thickness h, obtained from results of measuring the particle-marker velocities directly adjacent to the film surface, is presented in Fig. 3a and b. The maximum surface velocity gradient corresponds to the minimum values of the film thickness, where the minimum values of U correspond to h^* (Fig. 2b) and can be computed by means of the Nusselt formula $U = gh^2/2\nu$ (line II in Fig. 3a), and the maximum values of U^* for the rolling waves correspond to maximum values of h^* (Fig. 2b) and are described by the empirical dependence $U^* = c = kh^*$, where k is a proportionality factor in [13].

Results of measuring the surface velocity, averaged with respect to the wavelength λ, are presented in the table. It is seen that only for large amplitude waves do the values of the dimensionless surface velocity deviate from 1.5.

A generalized velocity profile for waves of two types is constructed in the coordinates y/h, u/U in Fig. 4 by using the results of measuring the local surface velocity (Fig. 3a). Here the line I is constructed by means of (1) and it describes all the experimental data corresponding to the sections 1 in both kinds of waves.

Therefore, cnoidal and sinusoidal waves with a ±5% spread in the experimental points are described by a self-similar profile in the form (1).

In the case of rolling waves the velocity profile along the wavelength varies relative to a parabolic section 1, from one less filled in the section 2 (the domain II in Fig. 4) to one more filled in the section 3 (domain III). The maximum deviation from the parabolic law is 15%.

The sections 4 are singular domains in the wave for which this deviation will possibly reach substantially large values.

NOTATION

x, y, longitudinal and transverse coordinates, m; h, film thickness, m; u, v, longitudinal and transverse velocity components, m/sec; U, surface velocity; c, phase velocity, m/sec; λ, wavelength, m; Q, specific fluid mass flow rate, m^2/sec; g, free fall acceleration, m/sec^2; ν, kinematic viscosity, m^2/sec; Re = Q/ν, Reynolds number. Indices: 0, mean values; superscript asterisk, maximum value; and subscript asterisk, minimum value.

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